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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 2743

Unique Paper Code : 32355402 GC-4

Name of the Paper : GE-4 Numerical Methods

Name of the Course : Mathematics : Generic Elective for Honours

Semester : IV

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the *six* questions are compulsory.

Attempt any *two* parts from each question.

Use of Scientific Calculator is allowed.

1. (*a*) Explain significant digits and Local truncation error with examples. An approximate value of π is given by 3.1428571 and its true value is 3.1415926. Find the absolute and relative errors.

P.T.O.

(b) Explain the Bisection method for computing the roots of equation $f(x) = 0$. Perform three iterations of the Bisection method in the interval (1, 2) to obtain roots of the equation $f(x) = x^3 - x - 1 = 0$.

(c) Define rate of convergence. Determine the rate of convergence for the Secant method. 12

2. (a) Perform four iterations of the Regula-Falsi method to obtain a root of the equation :

$$f(x) = 3x + \sin x - e^x = 0.$$

(b) Perform three iterations of Newton's method to find the root of the equation $x^4 - x - 10 = 0$ and starting approximation as 1.5.

(c) Perform two iterations of Newton's method to solve the non-linear system of equations with initial approximation (1, 1) :

$$f(x, y) = x^2 + y - 11 = 0 \text{ and}$$

$$g(x, y) = x + y^2 - 7 = 0.$$

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3. (a) Solve the linear system $Ax = b$ using Gaussian elimination with pivoting :

$$A = \begin{bmatrix} 6 & 2 & 2 \\ 6 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix} \text{ and } b = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}.$$

(b) Starting with initial vector $(x, y, z) = (0, 0, 0)$ perform three iterations of Gauss-Seidel method to solve the following system of equations :

$$2x - y = 7, -x + 2y - z = 1, -y + 2z = 1.$$

(c) Explain Thomas Algorithm and solve the following Tridiagonal system $Ax = b$ using the Thomas Method :

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 3 & 3 \\ 0 & 3 & 10 \end{bmatrix}, b = \begin{bmatrix} 10 \\ 17 \\ 22 \end{bmatrix}.$$

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4. (*a*) Find the unique polynomial P(x) of degree 2 or less using Lagrange interpolating formula for the following data :

$$x = [1, 3, 4]$$

$$f(x) = [1, 27, 64]$$

Also, estimate P(1.5).

(*b*) Prove the following relations :

(*i*) $\mu = \sqrt{1 + \frac{\delta^2}{4}}$

(*ii*) $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{\left(1 + \frac{1}{4}\delta^2\right)}$.

(*c*) By use of Richardson extrapolation, find f' (3) using the approximate formula :

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h}$$

with h = 4, 2 and 1, from the following values :

x	$f(x)$
−1	1
1	1
2	16
3	81
4	256
5	625
7	2401

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5. (*a*) The following data represents the function $f(x) = e^x$:

x	$f(x)$
1	2.7183
1.5	4.4817
2.0	7.3891
2.5	12.1825

Estimate the value of $f(2.25)$ usign the Newton's forward difference interpolation and compare with the exact value.

(*b*) Obtain the piecewise linear interpolating polynomial for the function $f(x)$ defined by the given data :

x	$f(x)$
0	1
1	2
2	5
3	10

and interpolate at $x = 0.5$ and 1.5.

(*c*) Find the approximate value of :

$$I = \int_0^1 \frac{dx}{1+x}$$

using :

(*i*) Trapezoidal rule

(*ii*) Simpson's rule. 13

6. (*a*) Apply Euler's modified method to approximate the solution of the initial value problem and calculate $y(1.3)$ by using $h = 0.1$:

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}, \quad y(1) = 1.$$

(*b*) Apply mid-point method (R.K. Second order) to solve the initial value problem :

$$\frac{dy}{dx} = yx^3 - 1.5y$$

from $x = 0$ to 2 where $y(0) = 1$ by using $h = 1$.

(*c*) Apply finite difference method to solve the given problem :

$$\frac{d^2y}{dx^2} = y + x(x-4), 0 \le x \le 4,$$

$$y(0) = 0, y(4) = 0 \text{ with } h = 1.$$ 13